Thinking about information processing in biological systems

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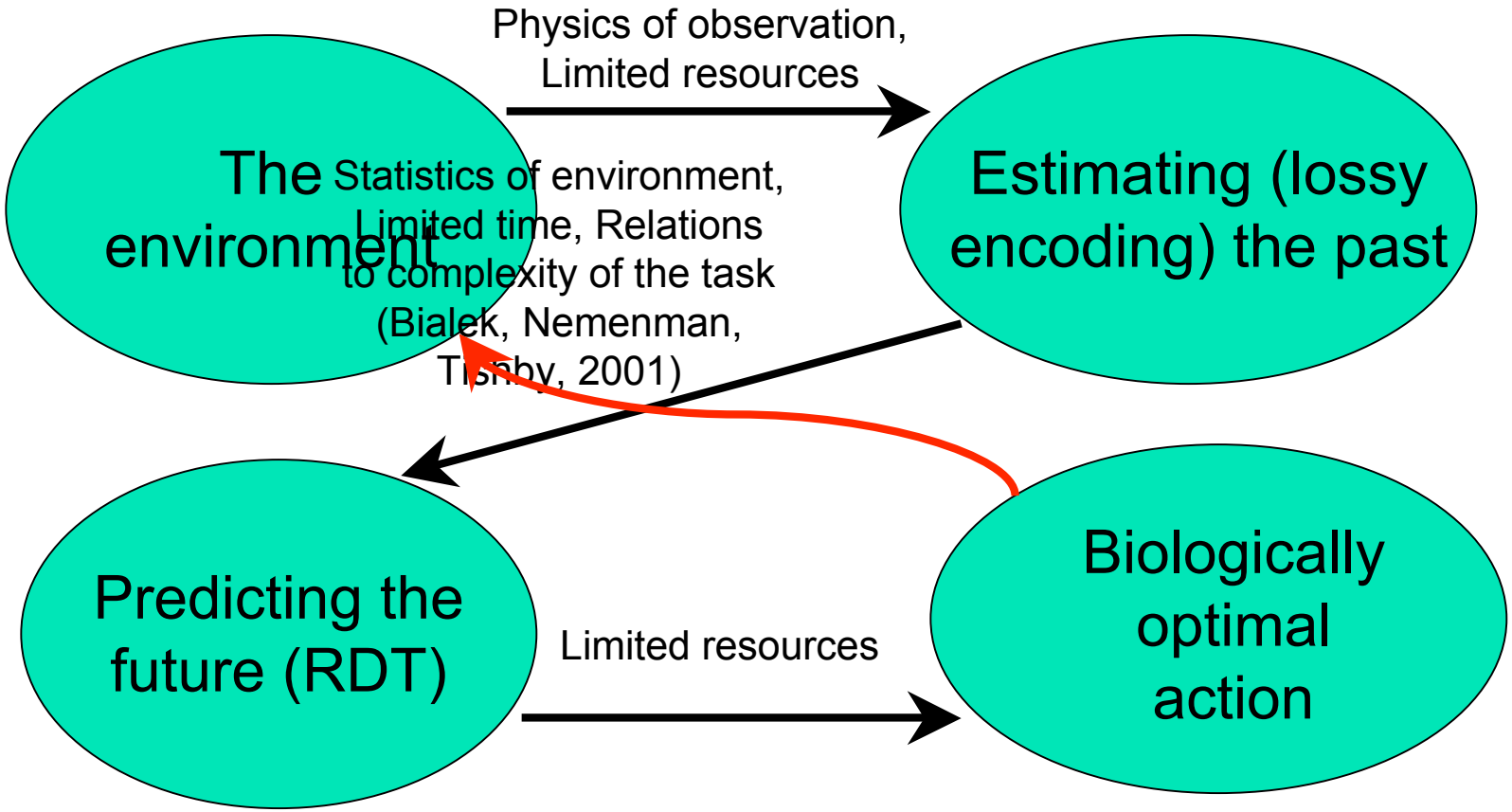
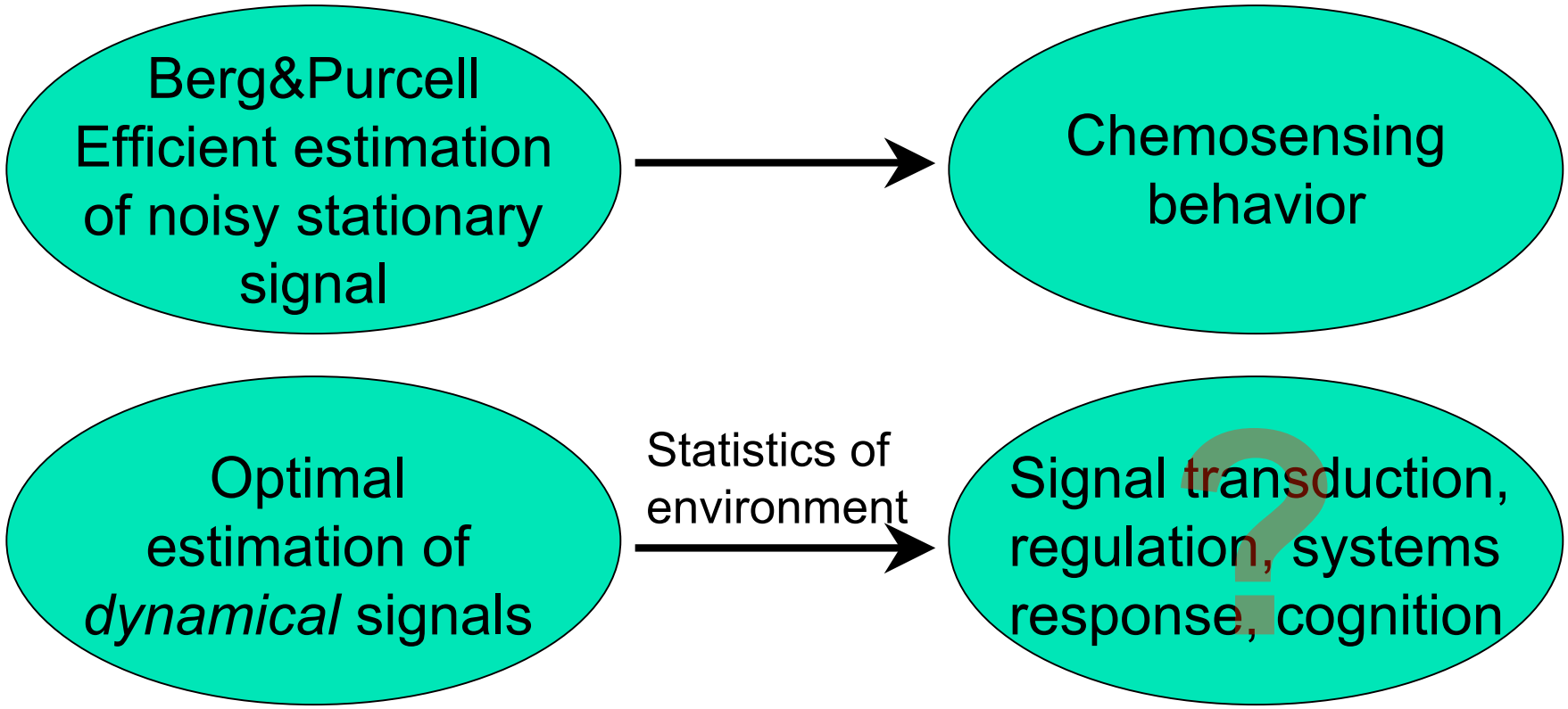
Life is…


Physics of observation,
Limited resources
The environment
Statistics of environment,
Limited time, Relations
to complexity of the task
(Bialek, Nemenman,
Tishby, 2001)
Estimating (lossy encoding) the past
Predicting the future (RDT)
Limited resources
Biologically optimal action

Efficient estimation as a biological design principle

- Berg and Purcell (1977). Chemosensing precision and reliability is limited by physical noise sources.
- Since then: single photon responses, transcription, chemotaxis run length, motion estimation,… - all are at physical limits to sensing.
- The second arrow? (estimation of and reaction to a *dynamical* environment).

In time learning/prediction: *necessary* for active response

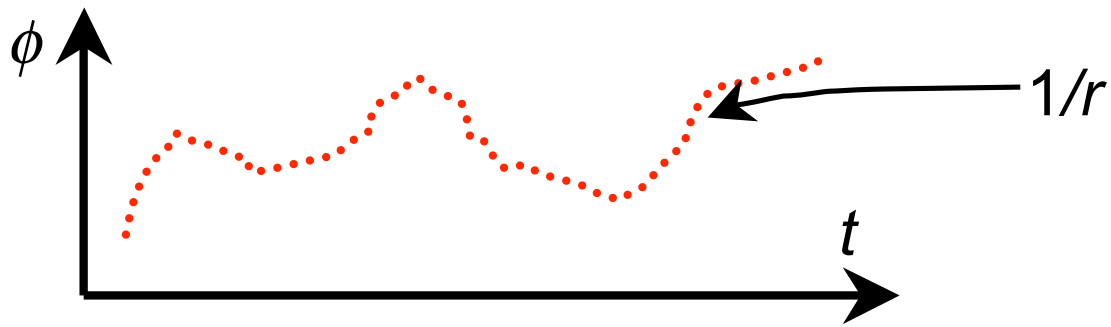


Mathematics of prediction: A limited form of prediction

- Estimation of dynamical signal “right now” (t=0) from observations of its past (t<0).
- Need to know time statistics of the signal.

Mathematics of prediction: A limited form of prediction

For a signal ϕ sampled at rate r and with $C(t) \xrightarrow[t\to 0]{} \sigma^2\left(1-\left(\frac{t}{\tau}\right)^{\nu}\right)$



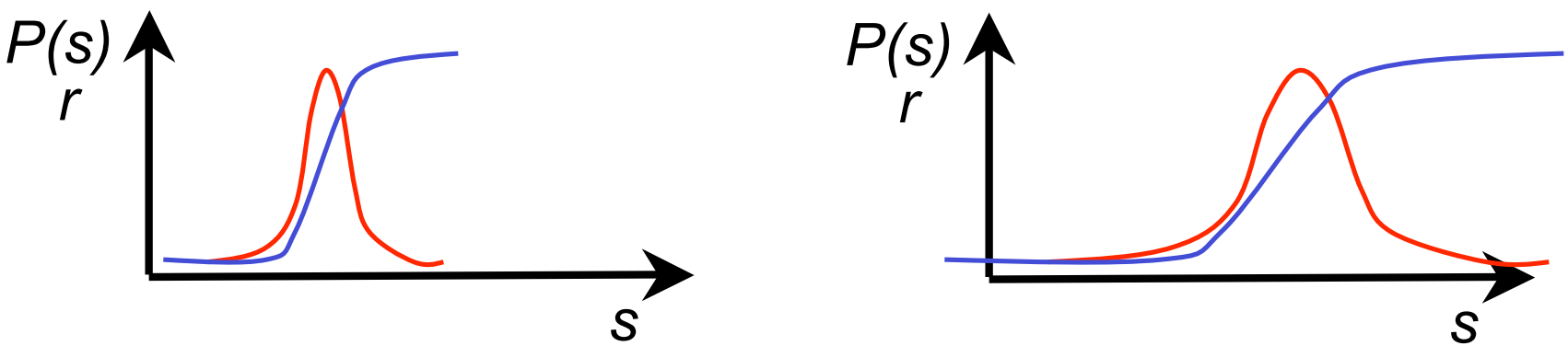
$$\left\langle \delta^2\phi \right\rangle \sim \frac{\text{noise}}{rt} + \left(\frac{t}{\tau}\right)^{\nu} = \min \quad \rightarrow \quad t \sim \left(\frac{\tau^{\nu}}{r}\right)^{1/(\nu+1)}$$

Var of the mean

Var of decorrelation

A note on optimal information transmission

$$P(s,r) \rightarrow I[P] = \left\langle \log \frac{P(s,r)}{P(s)P(r)} \right\rangle$$

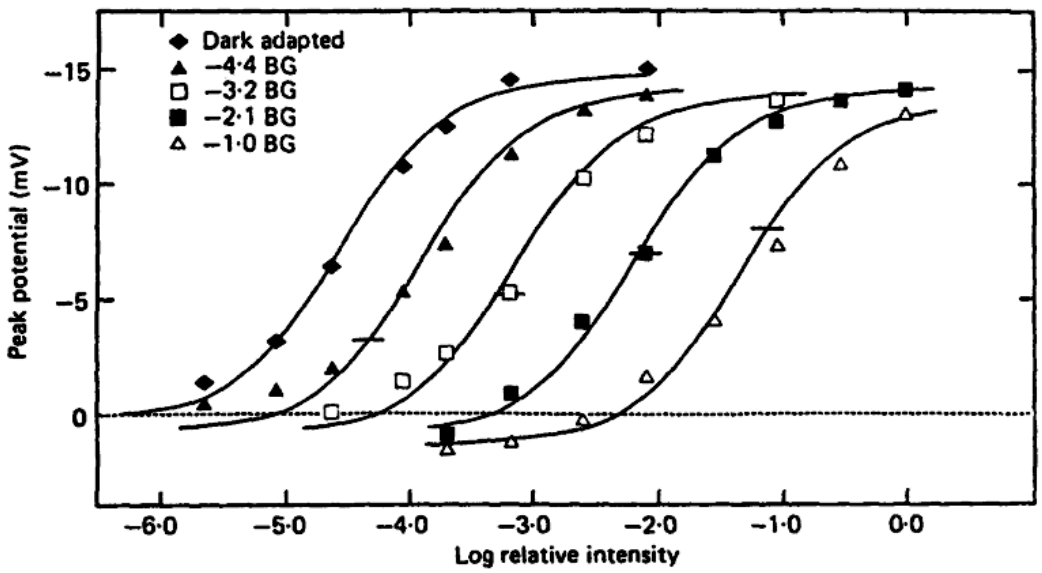


Matching mean and variance maximizes information transmission.

(Laughlin, 1981)

Turtle cone background light intensity adaptation

$$P(I) \propto \exp\left[-\frac{1}{2\sigma^2}\left(\log\frac{I}{I_0}\right)^2\right]$$



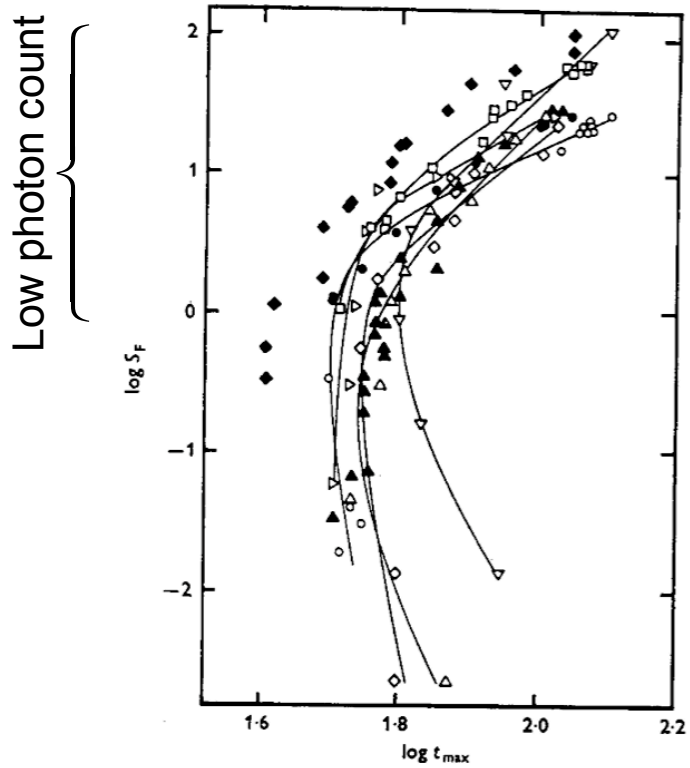
(Normann & Perlman, 1979)

Bckgr, log I	Adap, log I
dark	-4.4
-4.4	-3.8
-3.2	-3
-2.1	-2.3
-1	-1.3

$$I_a \propto I_0^{0.73}$$

Bad!

Response time adaptation



(Baylor & Hodgkin, 1974)

$$\tau \propto I_0^{1/3 \cdots 1/5}$$

Other animals range from 1/2 to 1/7.

$$I_a \propto I_0 \tau$$

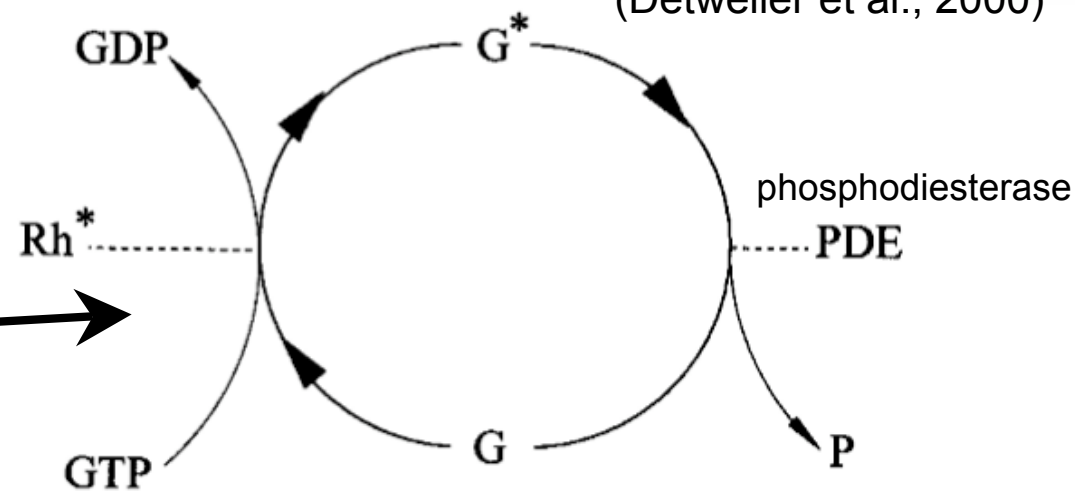
Probably not a coincidence:
Adapting to integrated flux.

What should τ be?

Cone: 3 low pass filters (at least):

- $\gamma + Rh \rightarrow Rh^*$
- $Rh^* \rightarrow PDE^*$
- $PDE^* \rightarrow GC$

(Detweiler et al., 2000)



$$\tau_R \frac{d\delta R}{dt} = -\delta R + g_R\left(\phi(t) + \eta_\phi(t)\right), \quad \phi = \log\frac{I}{I_0}, \left\langle \eta_\phi(t)\eta_\phi(0)\right\rangle = 1/I_0\delta(t)$$

$$\tau_P \frac{d\delta P}{dt} = -\delta P + g_P\left(\delta R + \eta_R(t)\right), \qquad \ldots$$

$\ldots$

Linear due to Ca feedback!

Solution
(for signal-limited precision)

$$I_0 \equiv I[\phi(t=0); v(t=0)] = \log \frac{\langle \phi^2 \rangle}{\langle \phi^2 \rangle - \frac{\prod g_i^2 \langle \phi_f^2 \rangle^2}{\langle v^2 \rangle}}$$

Note that this is not same as

$$I[\phi(t); v(t)] = \lim_{T \to \infty} \frac{1}{2T} \int_{-T/2}^{T/2} \frac{d\omega}{2\pi} \log\left(1 + SNR(\omega)\right)$$

which is the channel capacity.

Solution
(for signal-limited precision)

$$I_0 \equiv I[\phi(t=0); v(t=0)] = \log \frac{\langle \phi^2 \rangle}{\langle \phi^2 \rangle - \frac{\prod g_i^2 \langle \phi_f^2 \rangle^2}{\langle v^2 \rangle}}$$

Can also maximize total predictive information:

$$I\left[\phi(t>0); v(t \le 0)\right]$$

Finding τ

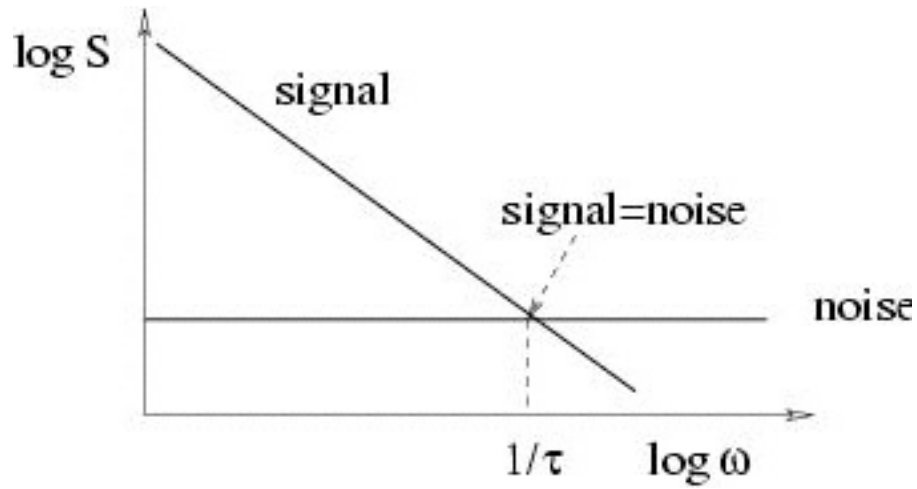
Maximize I_0 *w.r.t* τ

For:

$$S_\phi(\omega) \xrightarrow[\omega\to\infty]{} \omega^{-\alpha}$$

get:

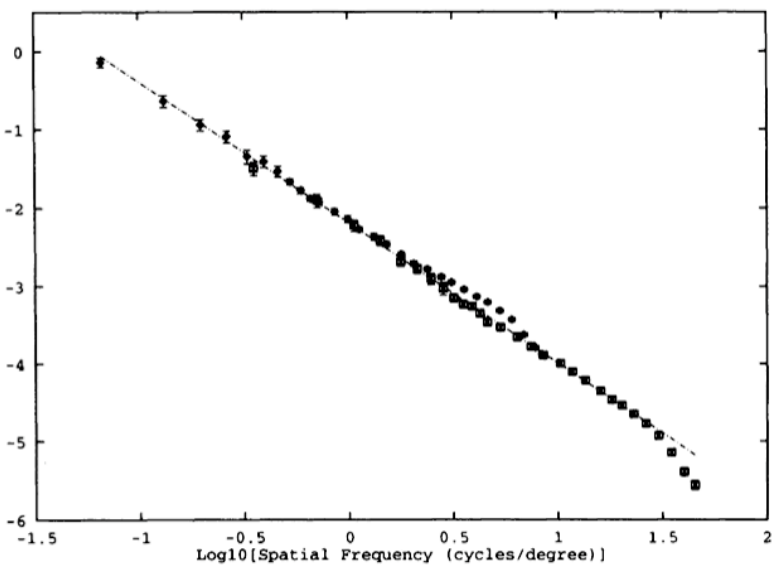
$$\tau \sim I_0^{-1/\alpha}$$



Best possible matched filter
(limited by biochemical mechanisms)

Also predicted by variance balance argument.

A problem



(Ruderman & Bialek, 1994)

- $1/k^{2-\varepsilon}$ spatial spectrum
- ~10 phoreceptors/fixation drift
- $1/\omega^{2-\varepsilon}$ temporal spectrum
- Should have $\tau \sim I_0^{-1/2}$

Wrong! But…

Structural constraint

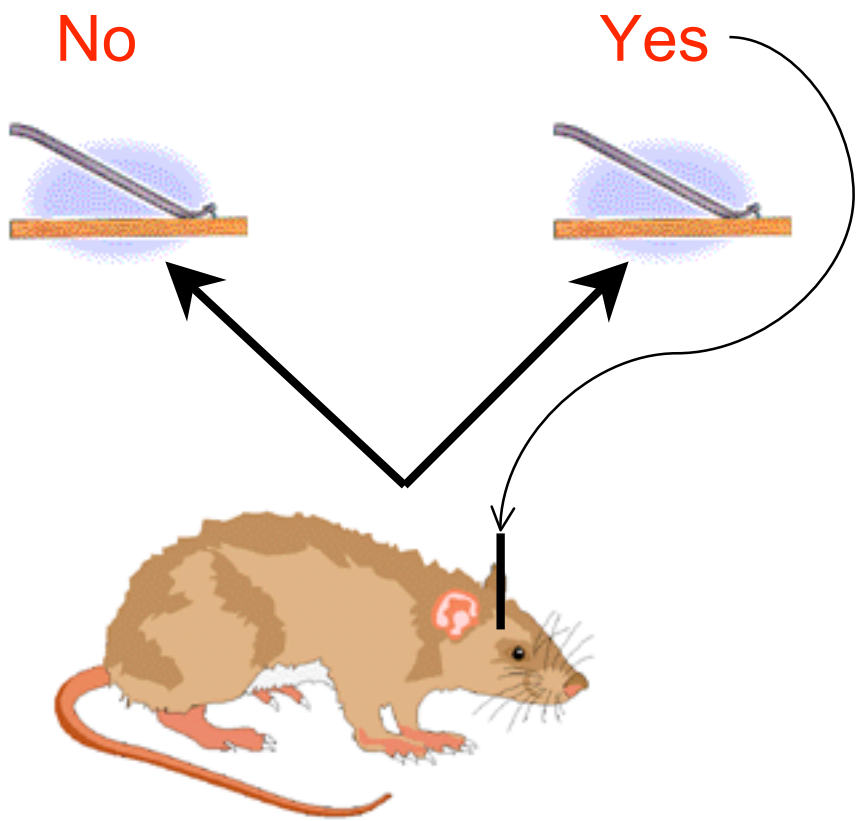
Rh* is the signal (for the adapting rest of the circuit), its temporal response is uncontrollable
(and badly known - Rieke & Baylor, 1998)

$$S_{Rh*} \sim \frac{1}{\omega^2} S_\phi \sim \frac{1}{\omega^4}$$

Given this signal, the rest of the biochemistry should adapt in agreement with experiment

$$\tau \sim I_0^{-1/4}$$

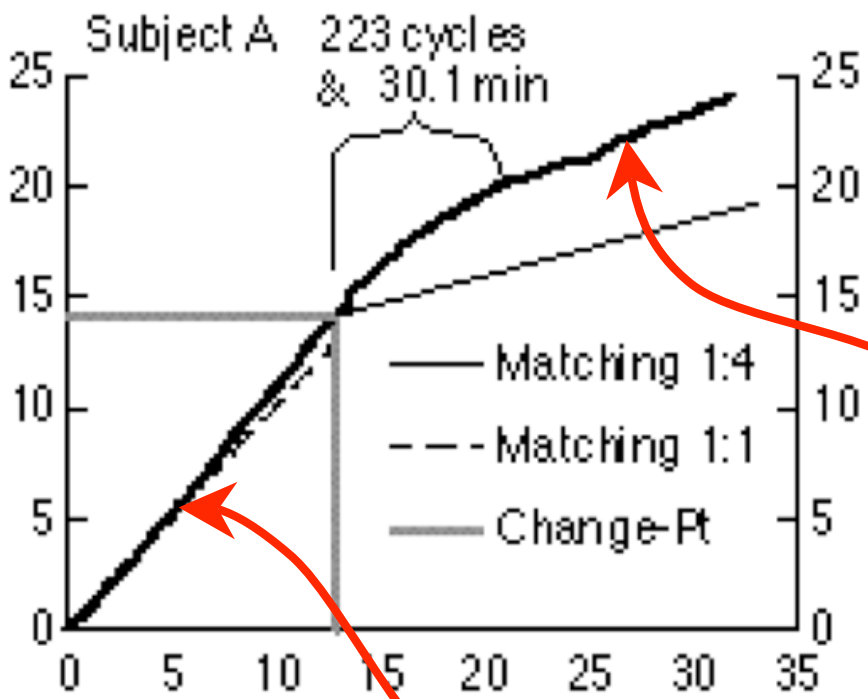
Rat matching experiments



(with Gallistel)

- Poisson deposition of rewards
- Rewards do not accumulate
- Possibly variable rate
- Changeover delay
- Rat matches

Rat matching experiments



(Gallistel et al 2001)

- Poisson deposition of rewards
- Rewards do not accumulate
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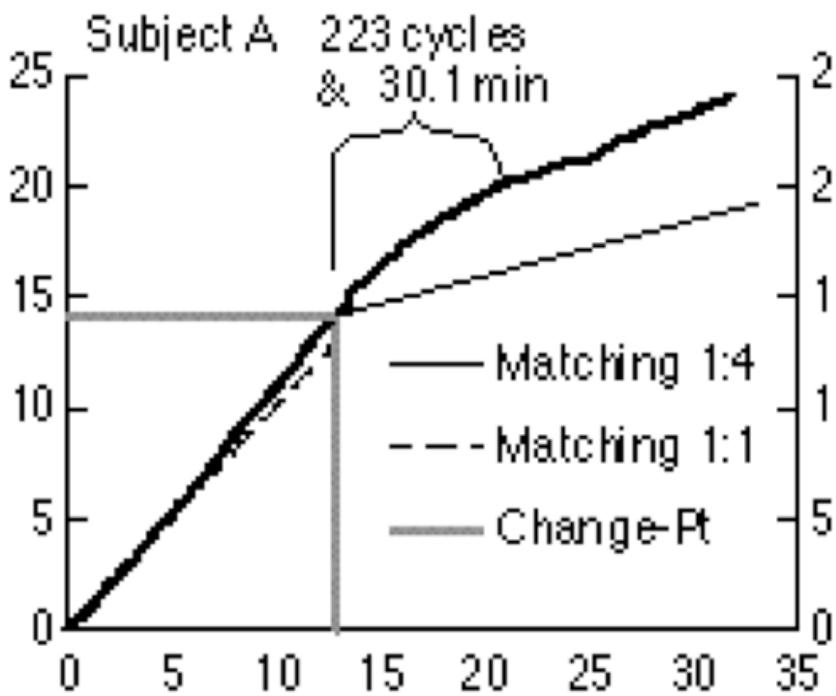
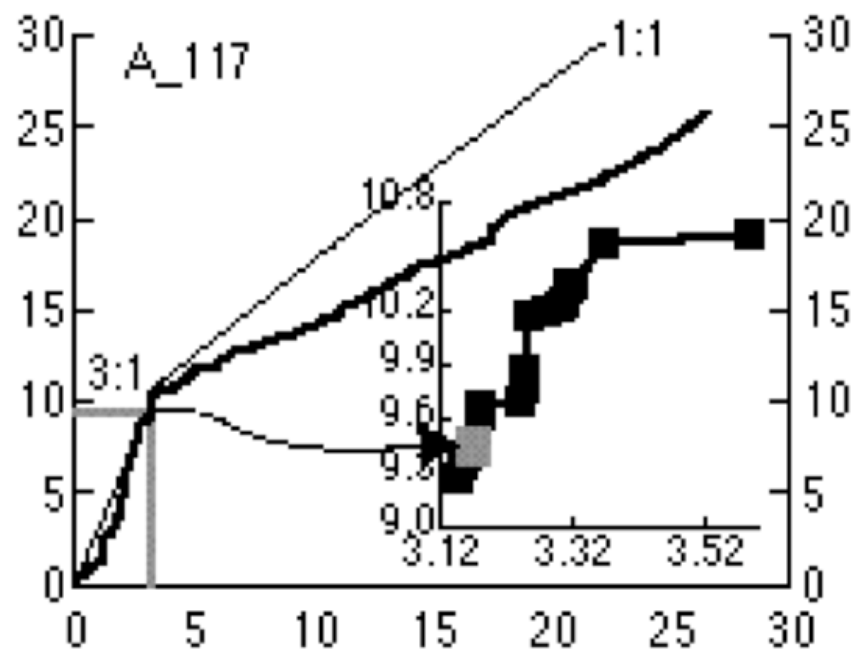
But: Time scales are history dependent. Can we explain?



$\tau = 30 \ldots 100 + \text{min}$



$\tau = 1 \ldots 2\,\text{min}$

(also note imperfect matching)

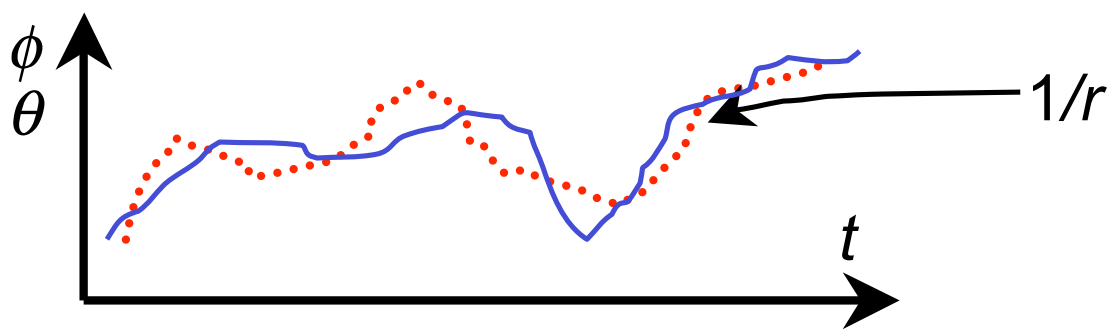
Optimal estimation: Bayes theorem

$$P[\phi(t) \mid \text{data}] = \frac{P\left(\text{data} \mid \phi(t)\right) P[\phi(t)]}{Z}$$

$$P[\phi(t)] \propto \exp\left[-\frac{l}{2}\int (\partial_t \phi)^2 dt\right], \quad \phi(t) = \log r(t)$$

Sampling

Sampling from a target $\theta(t) \to \phi_i \quad \text{at} \quad t_i$



Log-likelihood

$$P[\phi(t) \mid \{x_i\}, \{t_i\}] \propto \exp\left[-\frac{l}{2} \int dt\, (\partial_t \phi)^2 - \sum_i V(\phi_i - \phi(t_i)) \right]$$

Evolution in a random (time and space) potential

Examples of log-likelihoods

Poisson process:

$$\sum_i V_i = \int dt\, e^{\phi(t)} - \sum_i \phi(t_i) \quad \rightarrow \quad \int dt\, e^{\phi(t)} - \int dt\, e^{\theta(t)}\, \phi(t)$$

Learning a fluctuating mean of a Gaussian:

$$\sum_i V_i = \sum_i \frac{1}{2\sigma^2}\left(\phi(t_i) - \phi_i\right)^2 \quad \rightarrow \quad \frac{1}{2\sigma^2}\int dt\, r(t)\left(\phi - \theta\right)^2$$

In general, for fast sampling rate:

$$\sum_i V_i \quad \rightarrow \quad \int dt\, r(t)\, V\left(\phi - \theta\right)^2 \quad \rightarrow \quad r\int dt\, V\left(\phi - \theta\right)^2$$

A better solution (WKB): Learning a Poisson variable

Bialek, Callan, & Strong, 1996, Nemenman and Bialek, 2002

$$\tau = \sqrt{l / e^{\theta}}, \qquad \left\langle \delta^2 \phi_0 \right\rangle \approx \frac{1}{4} \frac{1}{\sqrt{e^{\theta} l}}$$

Time scales

Correlation time: $\tau \propto \sqrt{l / r}$

For stable period $(\tau_0 \sim 1\,\mathrm{hr})$:

$$l \approx 3 \cdot 10^6\, s, \quad r \approx 1/10 s^{-1}, \quad \tau \sim 5 \cdot 10^3\, s \approx 1.5\,\mathrm{hrs}$$

For variable schedule $(\tau_0 \sim 1\,\mathrm{min})$:

$$l \approx 1800\, s, \quad r \approx 1/10 s^{-1}, \quad \tau \sim 130\, s \approx 2\,\mathrm{min}$$

For monkeys (Sugrue et al, 2004) $(\tau_0 \sim 17\,\mathrm{samples})$:

$$l \approx 300 / r, \quad r, \quad \tau \sim 15\,\mathrm{samples}$$

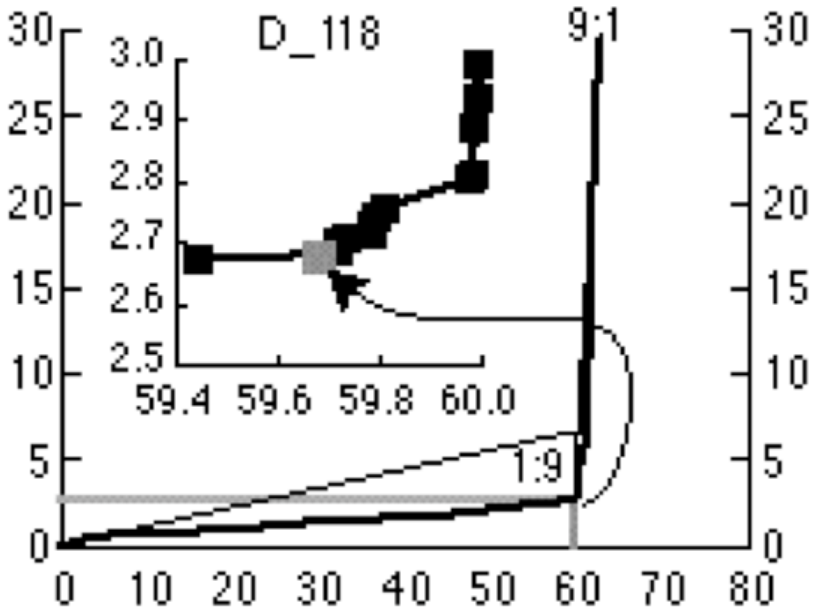
Importantly, estimate starts to change immediately in both cases

Self-consistent estimation of *l*

Averaging over $P[l]$ leads to correct estimation of the smoothness scale for fixed l (Nemenman and Bialek, 2002).

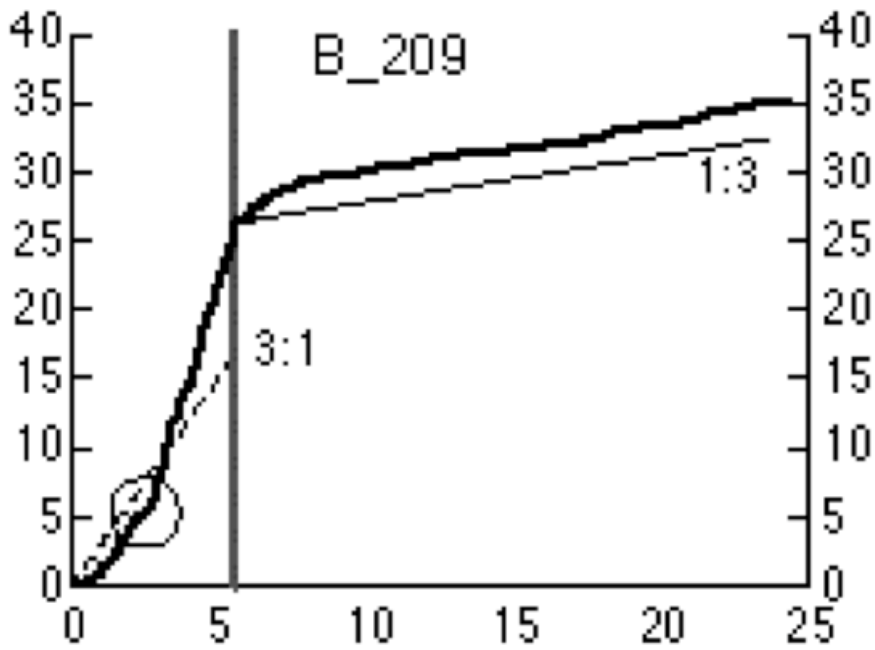
Can do the same for dynamic l.

Phenomenology: Abrupt changes



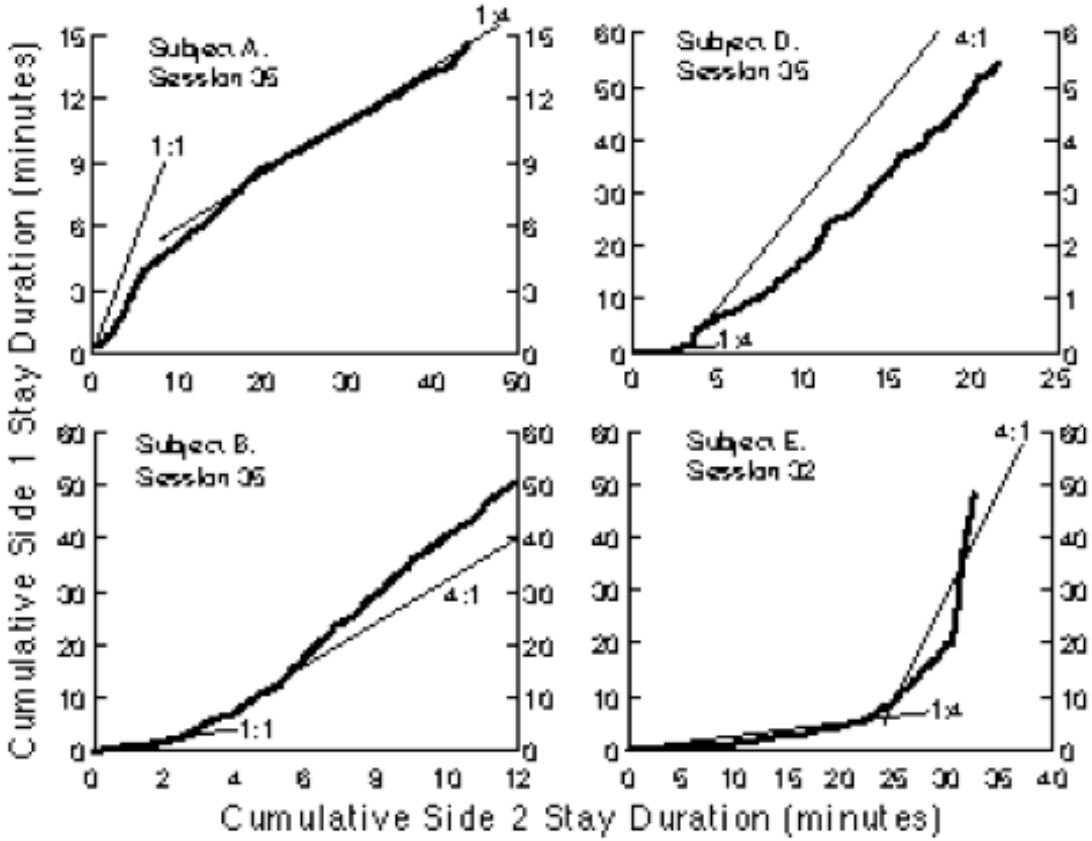
- Only after a few changes have been experienced
- Common during fast changes epochs

Phenomenology: Abrupt changes

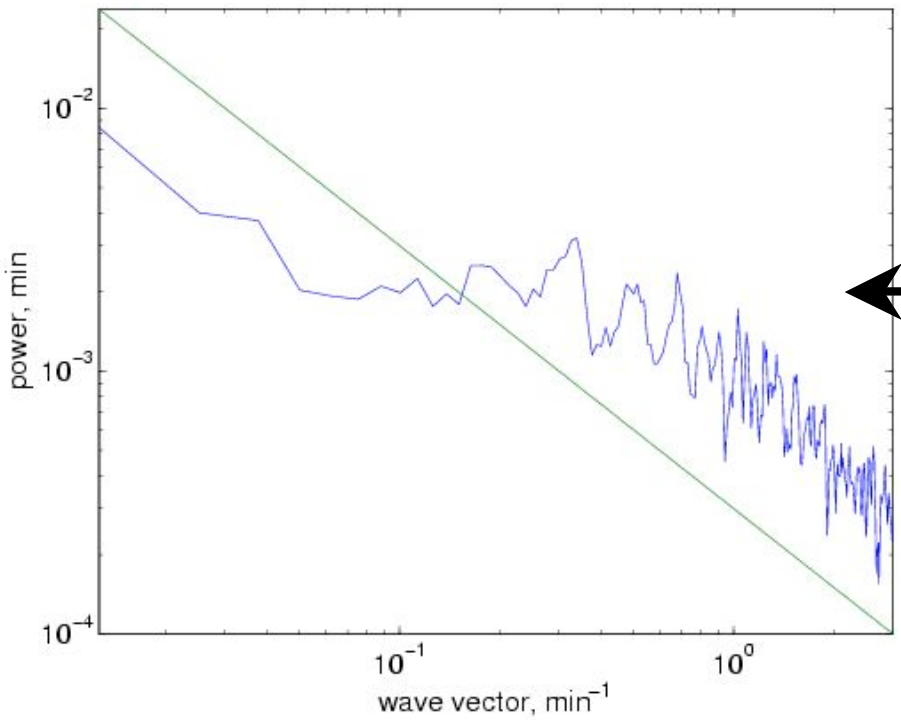


- Only after a few changes have been experienced
- Common during fast changes epochs
- (Metastable states)?

Phenomenology: Reversal to status quo ante

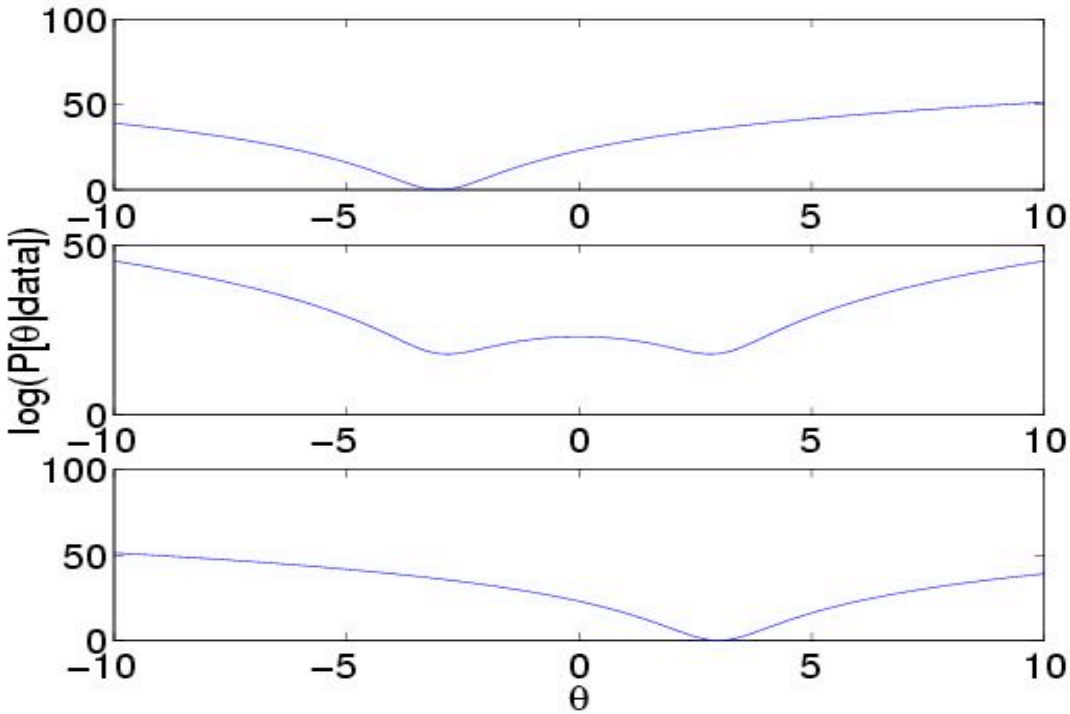


Caused by memory



- Overestimation of rate immediately leads to higher rate and persists
- Power spectrum of reward histories
- Two regimes clearly seen
- Peak at 0 - long range correlations

Abruptness, two time scales, and non-Gaussianity



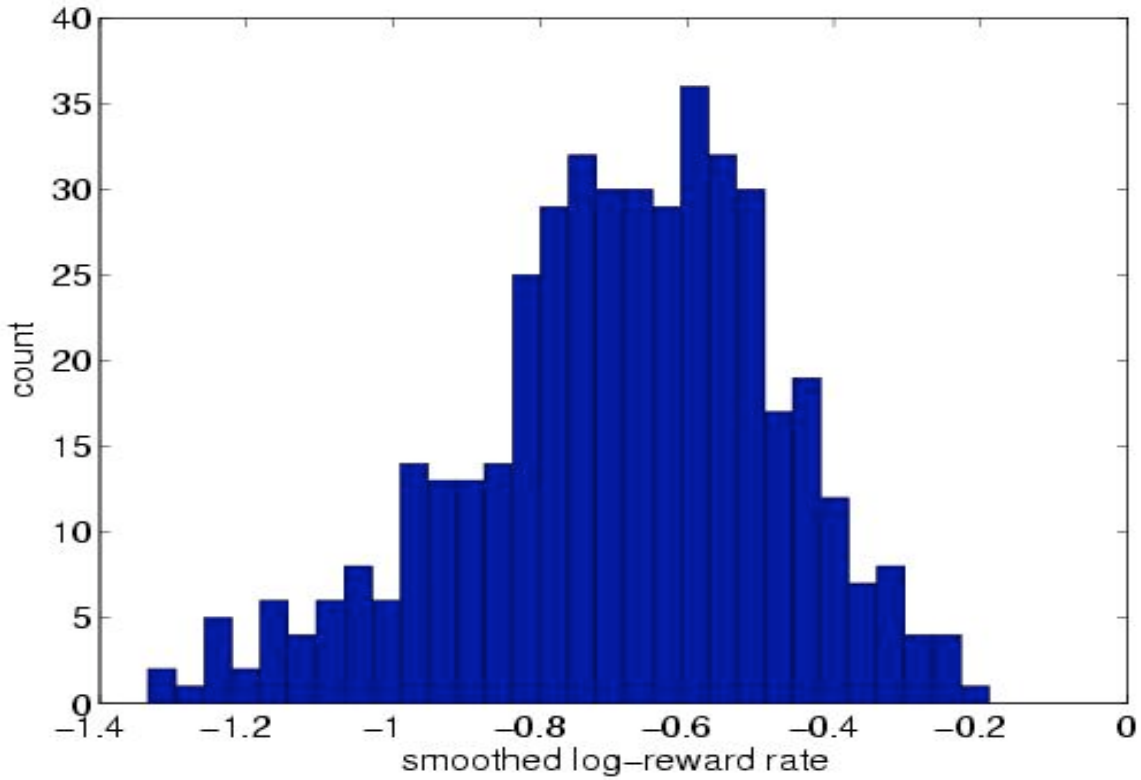
Critical periods?

Modeling memory: two scales and rapid readjustments

- Need memory scale (long), and adaptation scale (short)
- Signal changes on long time scale, while effects of self-perpetuation of rate are on shorter scales

$$P[\phi,\psi \mid \text{data}] \propto \exp\left[-\frac{\lambda}{2}\int dt\,(\partial_t\psi)^2 - \int dt\,W(\phi-\psi) - \cdots \right.$$
$$\left. -\frac{l}{2}\int dt\,(\partial_t\phi)^2 - \sum_i V(\phi_i - \phi(t_i))\right]$$

Non-Gaussianity of rate distribution



Dynamics of fast field

The field undergoes small fluctuations (zero point and due to irregular sampling) near the minimum of the energy

$$l\frac{d^2\phi_0}{dt^2} = 0 = e^{\theta}\frac{\partial V(\phi_0 - \theta)}{\partial \phi_0} + \frac{\partial W(\phi_0 - \psi)}{\partial \phi_0}$$

Two minima

For long-tailed distributions, will have two minima

$$\phi_0^{(1)} = \psi + \varepsilon_1(\theta - \psi)$$
$$\phi_0^{(2)} = \theta + \varepsilon_2(\psi - \theta)$$

Barrier depends on the shape and vanishes as

$$\psi \to \theta$$

Jumps possible from a metastable state near ψ to new minimum near θ.

Adiabatic dynamics of the slow field

$$P[\psi \mid \text{data}] \propto \exp\left[-\frac{\lambda}{2}\int dt\,(\partial_t \psi)^2 - \int dt\, W(\phi_0^{(i)}(t) - \psi(t))\right]$$

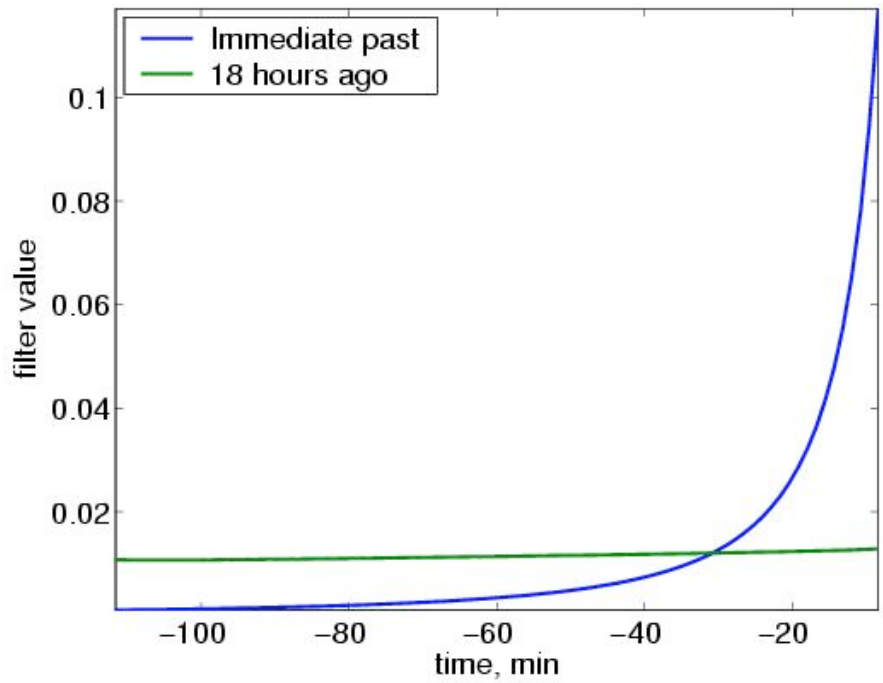
Either self-perpetuates, or approached the true solution (depending on which minima the fast field is in).

Critical periods?

Modeling reversals: long range correlations

Bialek & Zee, 1990 - Best estimate of ϕ is approximated by

$$\phi(t) = \sum_{t_i < t} F(t_i - t)$$



Optimal *F(t)* for a Gaussian process with $C \sim t^{-2}$ for a range near *t=0* and *t=18*hrs (normalized within the window).

Long-tailed filters explain reversal

- At the end of the session, rate estimates are effected mostly by the last (post-change) observation
- After a long delay, pre-change and post-change observations are almost equally weighed, but there are much more of the former.
- Wouldn’t work for exponential filters as used by Sugrue et al, 2004.
- Experiments to measure *C(t)* are now done.

Why matching?

- Matching is almost optimal for maximizing reward.
- Matching is almost optimal for tracking rate changes.
- Can it be that the bit value of a reward is higher than its food value? (Rats are curious!)
- Preliminary report: matching for accumulating rewards. Planning experiments to test matching to neutral stimuli.

Take home message:

- Optimal estimation of dynamic world seems to explain phenomenology from molecular scales, to cognitive psychology scales.
- Preliminary experimental comparisons.
- Better experiments are being done / are sought.
- For molecular networks, relation of phenomenology to structure waits to be analyzed.