

# Knowledge Representation Issues in Semantic Graphs for Relationship Detection\*

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## Abstract

An important task for Homeland Security is the prediction of threat vulnerabilities, such as through the detection of relationships between seemingly disjoint entities. A structure used for this task is a *semantic graph*, also known as a *relational data graph* or an *attributed relational graph*. These graphs encode relationships as *typed* links between a pair of *typed* nodes. Indeed, semantic graphs are very similar to semantic networks used in AI. The node and link types are related through an *ontology* graph (also known as a *schema*). Furthermore, each node has a set of attributes associated with it (e.g., “age” may be an attribute of a node of type “person”). Unfortunately, the selection of types and attributes for both nodes and links depends on human expertise and is somewhat subjective and even arbitrary. This subjectiveness introduces biases into any algorithm that operates on semantic graphs. Here, we raise some knowledge representation issues for semantic graphs and provide some possible solutions using recently developed ideas in the field of complex networks. In particular, we use the concept of transitivity to evaluate the relevance of individual links in the semantic graph for detecting relationships. We also propose new statistical measures for semantic graphs and illustrate these semantic measures on graphs constructed from movies and terrorism data.

## Introduction

A semantic graph is a network of *heterogeneous* nodes and links. In contrast to the usual mathematical description of a graph, semantic graphs have different types of nodes, and in general, different types of links. Also called attributed relational graphs (Coffman, Greenblatt, & Marcus 2004) and relational data graphs (used in the knowledge discovery literature), it is clear that the power of these graphs lies not only in their structure but also in the semantic information that resides on their nodes and links. Examples of semantic graphs

include citation networks where the nodes do not simply consist of papers, but also consist of authors, institutions, journals, and conferences. Another example is the Internet Movie Database where the nodes may be persons (actors, directors, etc.), movies, studios, and awards, among others. In Homeland Security, these graphs are used in a variety of information analysis tasks (Jensen, Rattigan, & Blau 2003; Coffman, Greenblatt, & Marcus 2004; Popp *et al.* 2004; Kolda *et al.* 2004). In particular, such graphs may be used for predicting threat vulnerabilities.

Data for semantic graphs come from relations parsed from text documents and/or data from relational databases. Our motivation for this work comes from our experience in constructing semantic graphs from two sources of data—movies data and terrorism data—to be discussed at the end of this paper. In both these cases, we were faced with a wide variety of choices: what are the node types, what are the link types, and how do these choices affect the algorithms that we intend to use on these graphs?

Several types of algorithms operating on semantic graphs are of interest to us. For example, to determine the nature of a possible relationship between two entities, a sub-graph consisting of the shortest paths (or another metric) between two nodes in the semantic graph may be constructed and examined (Faloutsos, McCurley, & Tomkins 2004). We refer to this process as *relationship detection*. Fast algorithms based on heuristic search (which improve on breadth-first search or bi-directional search) are available for this task, which either use or do not use the semantic information in the graph (Eliassi-Rad & Chow 2004; Chow 2004). These algorithms, however, depend on knowing which links (or link types) in the semantic graph are useful for detecting relationships. For example, two people who share a connection to “San Francisco” because they were born there are unlikely to have any real-life connection. One of the goals of this paper is to present automatic algorithms for determining which are useful links for relationship detection, as well as present concepts to help answer related questions.

In the past few years, a new field called *complex networks* (see, e.g., Albert & Barabasi (2002) and Newman (2003)) has emerged to study the structure of real-world networks. Statistical tools for characterizing graphs and networks have been developed, with the impetus of understanding the re-

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relationship between the structure and function of networks. Computer techniques have allowed these statistical measurements to be performed on very large real-world networks. In this paper we generalize some of these techniques in order to apply them to semantic graphs. For example, some types of nodes in semantic graphs can be connected to many other types of nodes, but generally have few actual links. We quantify this concept and hypothesize that nodes such as these are not useful for relationship detection. In addition, the concept of *transitivity* in social network analysis (called *clustering coefficient* in the complex networks literature) is useful for determining which are useful links for relationship detection.

In the following, we begin by describing semantic graphs and ontologies. We then use the concept of transitivity for evaluating links and link types for relationship detection. An important aspect of this paper is a presentation of new statistical measures for semantic graphs, as well as issues related to the scale (level of detail) of semantic graphs. Examples of semantic graphs for movies and terrorism data are given near the end of the paper.

### Semantic Graphs and Ontologies

A semantic graph consists of nodes and directed links, with each node having a *type* (e.g., movie). The set of types is usually small compared to the number of nodes. Each node is also labeled with one or more *attributes* identifying the specific node (e.g., *Shrek*) or gives additional information about that node (e.g., gross income). Links may also have types, for example, the (person  $\rightarrow$  movie) link may be of type ‘acted-in,’ or ‘directed.’ (In this case, multigraphs, or graphs that may have multiple links between the same pair of nodes, are possible.) In some semantic graphs, the meaning of a link between any two nodes is clear (although different between different pairs of node types), and no link types need to be defined. Finally, links may also have attributes. For additional details, see Sowa (1984).

Depending on the types of nodes and links and on the available information, certain relations can or cannot exist. The set of relations that can exist in a given semantic graph can be described by an auxiliary graph called an *ontology*, or a *schema* (Jensen & Neville 2002). More often, an ontology graph is created first by defining the types of relations that the semantic graph will encode. A small example of an ontology is given in Figure 1, showing three node types: person, meeting and city.

Special links in an ontology graph could describe *is-a* and *part-of* relationships among node types. This is a node type hierarchy that will be briefly mentioned when we discuss the scale of semantic graphs.

### Transitivity for Evaluating Nodes and Edges

Consider a node ‘San Francisco’ of type ‘city’ in a semantic graph, and suppose we have a database of people which includes city of birth among the data fields. A node ‘Alice’ of type ‘person’ may be linked to the node ‘San Francisco’ if Alice was born in San Francisco. Other nodes linked to node San Francisco imply a relationship to San Francisco

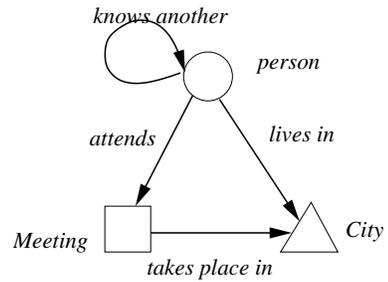


Figure 1: A small ontology consisting of three node types.

and in turn their relation to Alice. However, it is not clear that such relationships give useful information about Alice since most entities a short graph distance away from ‘Alice’ will have no real-life connection to Alice.

On the other hand, people born in a city such as ‘Tikrit,’ may have a much higher likelihood of knowing each other, that is, it may be important in this case to be able to associate two people through their city of birth. Instead of using a human with potential biases to evaluate nodes and links, an automatic procedure is desirable for objectively determining which nodes and links should be used in the semantic graph for relationship detection.

Another example is nodes of type ‘date.’ Dates could represent birthdates, dates of meetings, etc. For example, a node for a person born on 9-11-2001 may be linked to a node labeled ‘9-11-2001.’ However, two events sharing a date rarely predicts that two events are related. Our bias is to treat dates as attributes of nodes, rather than as its own node (with the type ‘date’). Topologically, a ‘date’ node may be connected to many other *types* of nodes, but generally each date node is connected to only a small number of other nodes. This may be an unbiased indication that a date is not useful for relationship detection.

### The transitivity concept

The concept of link transitivity is useful to address some of the above issues. If a node  $i$  has a link to node  $j$  and node  $j$  has a link to node  $k$ , then a measure of transitivity in the network is the probability that node  $i$  has a link to node  $k$ . In social networks and many other networks categorized as *small-world* networks, this probability is high. This is natural in social networks because a friend of a friend is also a friend in proportion that is much higher than in a random network. In general, we refer to  $j$  as a *neighbor* of  $i$  if  $i$  and  $j$  are directly connected in a graph. Also, we refer to the *degree* of a node as the number of neighbors it has.

The concept of transitivity is quantified as follows. The *clustering coefficient* of a node, denoted by  $C(i)$ , is a measure of the connectedness between the neighbors of the node. Let  $k_i$  denote the degree of node  $i$ , and let  $E_i$  denote the number of links between the  $k_i$  neighbors. Then, for an undirected graph, the quantity (Watts & Strogatz 1998)

$$C(i) = \frac{E_i}{k_i(k_i - 1)/2} \quad (1)$$

is the ratio of the number of links between a node’s neighbors to the number of links that can exist. We define  $C(i)$  to be 0 when  $k_i$  is 0 or 1. When  $C(i)$  is averaged over all nodes in the graph, we have the clustering coefficient for a graph. Note that high average clustering coefficient does *not* imply the existence of clusters or communities (subgraphs that are internally more highly connected than externally) in the graph.

### Relevance of a node

We consider the problem of determining whether a node in a semantic graph (e.g., “San Francisco” in a previous example) is useful for relationship detection. Consider a node  $i$  which has links to many other nodes. For now, we assume the links are of all the same type. To evaluate whether or not  $i$  is useful for relationship detection, we examine whether or not the neighbors of  $i$  are actually related in the semantic graph with high frequency. Whether or not two neighbors are related is decided by whether or not a link exists between the two neighbors. (A weaker condition if this does not hold is whether the two neighbors are linked via a third node which is already deemed a useful node for relationship detection.) This leads to the use of the clustering coefficient defined in Equation (1) to measure the relevance of a node  $i$  with degree greater than 1. The equation can be generalized so that  $E_i$  counts links with the weaker condition described above. A threshold  $\tau$  is needed and if  $C(i) > \tau$  then  $i$  is a useful node. If  $i$  is not a useful node, *all* the links involving  $i$  should not be used for relationship detection and could be removed from the semantic graph. If these links are removed,  $i$  could be made an attribute of the nodes that  $i$  originally linked to, in order not to lose any information.

The above can be generalized for semantic graphs when  $i$  is linked via many different types of links. In this case, instead of a count of relationships involving pairs of neighbors of  $i$ , a matrix  $M(t_1, t_2)$  is used instead. Here  $M(t_1, t_2)$  counts the number of relationships between pairs of neighbors  $(a, b)$ , where  $a$  is linked to  $i$  via type  $t_1$  and  $b$  is linked to  $i$  via type  $t_2$ . Small entries in this matrix gives *pairs* of link types (associated with  $i$ ) that should not be traversed in relationship detection.

### Relevance of a link

The relevance of an existing or potential relationship between two nodes  $a$  and  $b$  can be evaluated by how many neighbors they have in common. More precisely a relevance measure may be defined as

$$S(a, b) = \frac{|N(a, b)|}{|T(a, b)|} \quad (2)$$

where

$$N(a, b) = \{w \mid w \text{ is linked to } a \text{ and } b, w \neq a, w \neq b\}$$

and

$$T(a, b) = \{w \mid w \text{ is linked to } a \text{ or } b, w \neq a, w \neq b\}$$

with  $|T(a, b)| = \text{deg}(a) + \text{deg}(b) - |N(a, b)|$  where  $\text{deg}(a)$  is the degree of  $a$ . We have  $0 \leq S(a, b) \leq 1$  with large values of this relevance measure indicating a strong relationship

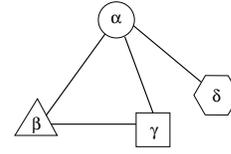


Figure 2: A particular ontology for which neighbors of  $\alpha$  of type  $\delta$  can never be connected to neighbors of type  $\beta$  or  $\gamma$ .

between  $a$  and  $b$  supported by a high proportion of common neighbors. This quantity is similar to the clustering coefficient and can be generalized to involve neighbors  $w$  farther from  $a$  and  $b$ .

There are many applications of this relevance measure. For example, pairs of nodes with no existing link can be evaluated to check if a latent link might exist. In another example, the relevance measure can be computed for all links of a given type. A low average of this relevance measure indicates that the given link type is not useful for relationship detection; there is not a strong relation between nodes incident on a link with the given type. A high relevance measure for a link when the average relevance measure for the link type is low (and vice-versa) indicates an outlier that may be interesting to investigate. This relevance measure must be used carefully, however, since it uses links that it assumes confers bona fide relationships.

It must also be recognized that a low relevance measure for an individual link does not imply that the link is unimportant. On the contrary, the notion of the “strength of weak ties” (Granovetter 1973) suggests that these links are critical in some sense. It is when almost all links of the *same* type have low relevance measure (and this link type is not a “secretly knows”  $b$ ) that this link type should not be used in relationship detection.

### Generalization of clustering coefficient for semantic graphs

The clustering coefficient defined earlier has little meaning for semantic graphs as it mixes different types of nodes and it does not include the constraints imposed by the ontology. To illustrate this, consider the ontology for a semantic graph given by Figure 2. In this case, a node of type  $\alpha$  can be connected to types  $\beta$ ,  $\gamma$  and  $\delta$ , but a neighbor of type  $\delta$  can never be connected to neighbors of type  $\beta$  or  $\gamma$ . In order to avoid unrealistically small values of the clustering coefficient we thus have to divide by the number of links actually *allowed* by the ontology and obtain

$$C(i; \alpha) = \frac{E_i}{E(i; \alpha)} \quad (3)$$

where  $E(i; \alpha)$  denotes the maximum number of links allowed by the ontology.

### Statistical Measures for Semantic Graphs

Along with clustering coefficient, two other relevant graph properties that have been developed for standard (non-semantic) graphs are *distributions of node degree* (number

of neighbors of a node) and *average path length* between any two nodes in the graph. Together, these three graph properties can be useful for studying the properties of a semantic graph for representing knowledge.

Many real-world networks have high clustering coefficient, much higher than  $O(1/n)$  for random graphs, where  $n$  is the number of nodes in the graph. We believe that properly constructed semantic graphs must also have moderately high clustering coefficients. Low values of clustering coefficient may indicate that the linkage information in the semantic graph is incomplete. Very high values of clustering coefficient may also indicate a poorly constructed semantic graph where all the nodes are very highly linked to each other (the limit is a fully connected graph), indicating little discrimination in how the nodes are connected.

The average path length,  $\ell$ , in a semantic graph must also not be too small (which is also associated with very high clustering coefficients). When the average path length is small, almost all nodes are approximately the same graph distance from each other, giving little discriminatory ability to path-length based algorithms for detecting relationships.

For example, an ontology graph may contain a node (e.g., a node of type ‘provenance’) to which every other node in the ontology is linked. In this case, the maximum shortest path length in the ontology graph is 2, which also suggests that the average path length in the semantic graph is small. It may be useful to identify nodes or links in the ontology graph that dramatically shorten the average path length. These nodes and links are potentially not useful for relationship detection.

The connectivity distribution  $P(k)$  is of interest for semantic graphs, particularly the existence of nodes with very high degree, as in the case of scale-free networks (Barabasi & Albert 1999; Amaral *et al.* 2000). In a relationship detection path search, paths through very high degree nodes are deemed less informative (Faloutsos, McCurley, & Tomkins 2004). For example, in a social network, two people who know a popular person are less likely to know each other; the linkages to the popular person should be disregarded in the relationship detection search since they may confer erroneous relationships.

It is believed that power-law connectivity distributions arise when there is little or no cost involved in the formation of links in the network (Amaral *et al.* 2000). Without this property, no nodes would be able to acquire a very large number of links. This may suggest that a graph with power-law degree distribution may contain many weak linkages. However, these weak linkages cannot be disregarded; Cf. strength of weak ties, mentioned above.

For semantic graphs, we showed above how to extend the concept of clustering coefficient. In the next subsections, we expand the potential usefulness of other concepts for semantic graphs.

### Extension of node degree

Even in the simple case of connectivity, a given value  $k$  of the connectivity of a node of type  $\alpha$  has no real meaning for semantic graphs. Indeed, as shown in Figure 3 the topological connectivity in both cases is  $k = 4$  but the meaning of it

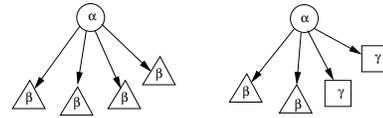


Figure 3: Two examples for which the  $\alpha$ -type node has topological connectivity  $k = 4$  but with a different meaning in each case, Cf. Jensen & Neville (2002).

is very different in each case.

In the first case, the environment is very homogeneous while it is not in the second case. Another complexity comes from the fact that the number of  $\beta$ -type nodes can be very large thus inducing a bias in the connectivity of the other nodes.

The ontology implies that each node of type  $\alpha$  can be connected to a certain number,  $k_\alpha^0$ , of other types. In the semantic graph, we have a total number of nodes  $n = \sum_\alpha n_\alpha$  and we denote the nodes by  $i = 1, \dots, n$ . The type of a node is given by the function  $t(i)$ . We denote by  $k_{\alpha\beta}(i)$  the number of neighbors of type  $\beta$  of a node  $i$  of type  $\alpha$ . The usual topological connectivity of the node  $i$  (which is of type  $\alpha$ ) is then given by

$$k_\alpha(i) = \sum_\beta k_{\alpha\beta}(i). \quad (4)$$

Using this quantity, we can define the average connectivity of type  $\alpha$  which is just the average over all nodes with type  $\alpha$  as

$$\overline{k_\alpha} = \frac{1}{n_\alpha} \sum_{i, t(i)=\alpha} k_\alpha(i). \quad (5)$$

If we want to compare the different types relative to their connectivity, it is important to remember that some types can be connected to many others (such as persons which can be linked to others persons, cities, meeting, jobs, etc.) while other types are only linked to one type (such as a conference which takes place only at one location). In order to compare the different types we thus have to rescale by the number of different neighbor types they can have according to the ontology:

$$m_\alpha = \frac{\overline{k_\alpha}}{k_\alpha^0}. \quad (6)$$

This quantity indicates the average number of neighbors *per type*. This quantity however does not tell us if there are large connectivity fluctuations or if in contrast all nodes of a given type have essentially the same connectivity. We thus have to measure the connectivity variance *per type* which is calculated using the second moment

$$\overline{k_\alpha^2} = \frac{1}{n_\alpha} \sum_{i, t(i)=\alpha} k_\alpha^2(i) \quad (7)$$

with the dispersion per type given by

$$\sigma_\alpha^k = \frac{[\overline{k_\alpha^2} - (\overline{k_\alpha})^2]^{1/2}}{k_\alpha^0}. \quad (8)$$

Another possible way to characterize the connectivity distribution per type is to plot the connectivity distribution. However, the dispersion around the average is already a first indication of the nature of the connections for different types. For some cases, the fluctuations will be small, while for others it can fluctuate greatly (such as the number of persons a person knows).

### Disparity of connected types

The above quantities tell us the expected number of connections of a node of a given type to another type but not the correlations between different types. Indeed, a type  $\alpha$  can preferentially link to a type  $\beta$  while it could be in principle also be linked to other types (as given by the ontology).

We thus quantify the disparity (or affinity) of each type to link to other types. In order to do this we use a convenient quantity—denoted by  $Y_2$ —which was introduced in another context (Derrida & Flyvbjerg 1987; Barthélemy, Gondran, & Guichard 2003). In order to understand the meaning of this quantity let us consider an object that is broken into a number  $N$  of parts, each part having a weight  $w_i$ . By construction  $\sum_i w_i = 1$  and  $Y_2$  is given in this case by

$$Y_2 = \sum_i [w_i]^2. \quad (9)$$

If all parts have the same weight  $w_i \sim 1/N$  then  $Y_2 \sim 1/N$  is small (for large  $N$ ). In contrast, if we have  $w_1 = 1/2$  and the rest is small implying  $w_{i \neq 1} \sim 1/2(N-1)$  then we obtain  $Y_2 \sim 1/4$ . This simple example can be easily generalized to more complicated situations and shows that a small value of  $Y_2$  indicates a large number of relevant parts while a larger value (typically of order  $1/m$  where  $m$  is of order unity) indicates the dominance of a few parts.

We now apply this idea to the number of types to quantify the disparity of a node or the affinity of a type. The quantity  $Y_2$  is first defined for a given node  $i$  of type  $\alpha$

$$Y_2(i; \alpha) = \sum_{\beta} \left[ \frac{k_{\alpha\beta}(i)}{k_{\alpha}(i)} \right]^2. \quad (10)$$

In order to get results with statistical significance, we average this quantity over all nodes of the same type and we also compute its dispersion  $\sigma_{\alpha}^Y$ :

$$\bar{Y}_2(\alpha) = \frac{1}{n_{\alpha}} \sum_{i, t(i)=\alpha} Y_2(i; \alpha), \quad (11)$$

$$\sigma_{\alpha}^Y = \left[ \overline{Y_2^2(\alpha)} - (\bar{Y}_2(\alpha))^2 \right]^{1/2}. \quad (12)$$

These results must however be weighted by the fact that some types are more numerous than others which could be a reason why they appear more often than others. For a given node  $\alpha$ , we denote by  $\mathcal{V}(\alpha)$  the set of types which can be connected to  $\alpha$  as given by the ontology. If a node has  $k$  neighbors, and if these neighbors are picked at random in the set of different nodes with population  $n_{\beta}$ , we then obtain a disparity given by

$$Y_2^r = \sum_{\beta \in \mathcal{V}(\alpha)} \left[ \frac{n_{\beta}}{n} \right]^2. \quad (13)$$

Again, this quantity will be very small if all types are uniformly present in the semantic graph  $Y_2^r \sim 1/N$  (where  $N$  is the total number of different types) and if it is of order unity then essentially a few types are over-represented. In order to take these heterogeneities into account it is thus necessary to rescale  $Y_2(\alpha)$  by  $Y_2^r$  and to form the factor

$$R(\alpha) = \frac{Y_2(\alpha)}{Y_2^r} \quad (14)$$

and its corresponding dispersion,

$$\sigma_{\alpha}^R = \frac{\sigma_{\alpha}^Y}{Y_2^r}. \quad (15)$$

A large value (larger than one) of  $R(\alpha)$  indicates that type  $\alpha$  preferentially links to a small number of types and that its neighbor types  $\mathcal{V}(\alpha)$  are diverse in number. If  $R \ll 1$ , the type  $\alpha$  may still be preferentially connected to a small set of types but the diversity of the numbers of each neighbor type is small.

The dispersion  $\sigma^R(\alpha)$  indicates whether the behavior as described by the average value  $R(\alpha)$  is typical, or if in contrast there is large diversity among the nodes of type  $\alpha$ .

Other usual quantities that are measured in order to characterize a large network can also be generalized without any difficulty. For example, degree distributions should be examined by type of node. In a semantic graph, the overall degree distribution may not be meaningful, but the degree distribution for a specific node type may be power-law, etc. As a further example, the average path length generalizes to become a matrix  $\ell_{\alpha\beta}$  where  $\alpha$  indicates the source node of the shortest paths while  $\beta$  is the target node. This matrix will in general have entries with very different values.

## Scale in Semantic Graphs

Given a knowledge base of relational data, the choice of ontology depends on what information needs to be captured in the semantic graph, and how easily certain information needs to be retrieved. The level of detail (or scale) chosen for the ontology (choice of node and link types) will have a direct impact on the properties of the corresponding semantic graph.

In the simplest ontology, we have nodes of only one type. In the example of the movies database, this ontology is a simple network of actors without any types and two actors are connected if they played in the same movie. At the next finer scale, we have actors and movies as node types. In this case, the ontology is an actor connected to a movie if he played in that movie. This is a special case of a semantic graph which is a *bipartite* network (two types of nodes, with links only between the two types). Coarser models lose some of the information present in finer models but can be useful for large-scale computations, such as multi-level search techniques.

At the finest scale of a terrorist network, we may have nodes of type ‘Religious Terrorist Organization’ and ‘Political Terrorist Organization.’ A coarser model may aggregate nodes of these two types into a new type, ‘Terrorist Organization’ (or the aggregation may occur directly if a type hierarchy is available). Depending on what information needs to

be preserved, it may or may not be important to distinguish between these two node types at the structural level of the semantic graph.

We note that in Homeland Security tasks, data analysis more often involves searching for outliers rather than commonplace patterns. Thus it is essential that the fine scale data is retained and the coarse scale data is used appropriately (for example, as an aid in managing and processing large-scale data).

### Effect of scale on statistical measures

Here we simply illustrate the effect of scale on the clustering coefficient. We consider a random bipartite graph with Poisson distributed numbers of both movies per actor (with average  $\mu$ ) and actors per movie (with average  $\nu$ ). We suppose that we have  $n_A$  actors and  $n_M$  movies and the fact that each link connects an actor to a movie imposes the constraint

$$\frac{\mu}{n_A} = \frac{\nu}{n_M}. \quad (16)$$

This model can be considered as a ‘‘hull’’ model since there are no particular correlations here. If one computes the clustering coefficient of the one-mode projection of this network, one obtains (Newman, Strogatz, & Watts 2001)

$$C = \frac{1}{\mu + 1}. \quad (17)$$

This quantity is finite even in the limit of very large networks  $n_{A,M} \rightarrow \infty$ . This is in contrast with the usual random network for which

$$C \sim \frac{1}{n} \quad (18)$$

where  $n$  is the number of nodes. At this stage the conclusion is that the actor network is very clustered and different from a random network with no correlations. This is however clearly an incorrect statement since the existence of a large clustering coefficient here is a consequence of the network construction procedure.

## Examples

### Movies data

The ‘‘Movies’’ test data at the UCI KDD Archive contains information about movies, persons (actors, directors, etc.), studios, awards, etc. The data was originally compiled by Gio Wiederhold (Stanford University). We used this data to construct an ontology and semantic graph to express most of the information in the dataset. Figure 4 shows the ontology graph that we developed. In the figure, the meaning of most of the links is obvious. However, the person-person link implies *married-to*, *lived-with*, or some other non-professional relationship; the person-studio link implies *founded*; the movie-movie link implies *sequel-to*. We note that the data is very incomplete.

In this ontology, the best meaning of the node Role is unclear. For example, are two actors linked to the same Role node in the semantic graph if they played the role of Villain in two different movies? Alternatively, a role node in the semantic graph may only link to actors playing a given role in a single movie. We arbitrarily chose the former in our case.

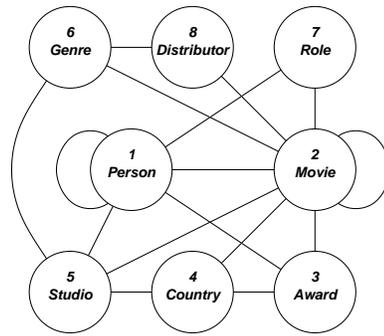


Figure 4: Movies ontology.

A related question, which is structurally similar but semantically different is the following. Should two actors who win a Best Actor award be linked to the *same* Award node in the semantic graph? In this case we did not choose this interpretation since it seems that awards are individual entities, whereas roles are not.

Table 1 summarizes the node types, frequencies, and other statistical measures for the movies semantic graph. The results show high dispersion of average connectivity per type, for all types. Further, the disparity of connected types is not particularly different from a random model. These indicate a relatively well-constructed semantic graph; there are no particular correlations (given the numbers of each node type) and thus the information content in the graph is high. The results will be very different for the terrorism data.

In the semantic graph, the nodes with the largest clustering coefficients depend on whether the types of the nodes are considered. In the standard case where the types are not considered, the node Maurice Barrymore has high clustering coefficient; the node is connected to Georgiana Drew Barrymore, Lionel Barrymore, Ethel Barrymore, etc., all of which are connected to each other. If node types are considered, then it is not important that neighbors of a node are not linked if they are not permitted to be linked according to the ontology. Now nodes that were missed with the above measure may have high clustering coefficient, e.g., the movie *Dogma* (perhaps due to the idiosyncrasies of the incomplete data).

In the semantic graph, the link between Columbia Pictures and drama (genre) has the most number of common neighbors (710). However, when the link relevance measure (Equation (2)) is used, which accounts for the number of links a node has, the link between Bud Abbott and Lou Costello is found (30 common neighbors). (We also found re-releases of movies under a new name in this process.) Further, a semantic version of relevance can be defined, which considers only the links that are allowed by the semantic graph. In this case, the link between Tokuma Studio and docu-drama is found. (Tokuma is linked to drama and the movie *Carences*; docu-drama is linked to *Carences* and Miramax; and Miramax is linked to drama.)

We also computed the average relevance per link type for the semantic graph. First, the link types of least fre-

| Node Type     | $n_\alpha$ | $m_\alpha$ | $\sigma_\alpha^k$ | $R(\alpha)$ | $\sigma_\alpha^R$ |
|---------------|------------|------------|-------------------|-------------|-------------------|
| 1 Person      | 21504      | 0.872      | 2.383             | 1.836       | 0.663             |
| 2 Movie       | 11540      | 1.131      | 0.816             | 1.299       | 0.644             |
| 3 Award       | 6734       | 2.579      | 10.201            | 0.905       | 0.144             |
| 4 Country     | 19         | 222.509    | 582.572           | 1.812       | 0.364             |
| 5 Studio      | 1075       | 1.948      | 9.534             | 1.241       | 0.408             |
| 6 Genre       | 39         | 77.803     | 160.060           | 0.512       | 0.154             |
| 7 Role        | 115        | 25.561     | 64.164            | 0.924       | 0.028             |
| 8 Distributor | 16         | 206.156    | 356.043           | 0.782       | 0.165             |

Table 1: Node types and statistics for the movies data: frequency of node type  $n_\alpha$ , average connectivity per type  $m_\alpha$  and its dispersion  $\sigma_\alpha^k$ , disparity of connected types  $R(\alpha)$  and its dispersion  $\sigma_\alpha^R$ . The results show high dispersion of average connectivity per type, for all types. Further, the disparity of connected types is not particularly different from a random model.

| Type                  | $n_\alpha$ | Type                   | $n_\alpha$ |
|-----------------------|------------|------------------------|------------|
| 1 Nation              | 92         | 31 Shooting            | 445        |
| 2 GeographicalRegion  | 85         | 32 Bombing             | 323        |
| 3 City                | 555        | 33 HostageTaking       | 14         |
| 4 Building            | 10         | 34 IncendDeviceAttack  | 18         |
| 5 Combustion          | 0          | 35 Lynching            | 3          |
| 6 Destruction         | 0          | 36 SuicideBombing      | 107        |
| 7 Device              | 0          | 37 CarBombing          | 114        |
| 8 GeographicArea      | 3          | 38 Arson               | 15         |
| 9 Government          | 1          | 39 HandgrenadeAttack   | 38         |
| 10 GovernmentPerson   | 2          | 40 Hijacking           | 15         |
| 11 Group              | 1          | 41 RocketMissileAttack | 14         |
| 12 Hole               | 1          | 42 KnifeAttack         | 53         |
| 13 Human              | 6          | 43 ChemicalAttack      | 9          |
| 14 JoiningAnOrg       | 0          | 44 LetterBombAttack    | 10         |
| 15 Killing            | 0          | 45 Stoning             | 3          |
| 16 OccupationalRole   | 3          | 46 VehicleAttack       | 7          |
| 17 Region             | 0          | 47 MortarAttack        | 8          |
| 18 SocialRole         | 1          | 48 Vandalism           | 4          |
| 19 StationaryArtifact | 1          | 49 Other               | 5          |
| 20 UnilateralGetting  | 0          | 50 Number              | 120        |
| 21 Vehicle            | 1          | 51 Continent           | 2          |
| 22 ViolentContest     | 1          | 52 GeneralStructure    | 6          |
| 23 Weapon             | 0          | 53 Month               | 12         |
| 24 Proposition        | 0          | 54 GeneralBuilding     | 2          |
| 25 BinaryPredicate    | 0          | 55 GeneralHuman        | 2          |
| 26 ForeignTerrOrg     | 28         | 56 Airbase             | 2          |
| 27 ReligiousOrg       | 0          | 57 Airport             | 3          |
| 28 TerroristOrg       | 53         | 58 State               | 4          |
| 29 Infiltration       | 8          | 59 Railway             | 1          |
| 30 Kidnapping         | 155        |                        |            |

Table 2: Node types and their frequencies,  $n_\alpha$ , for the terrorism data.

quency were Person-founded-Studio and Studio-located-in-Country. However, the links with lowest average relevance per link were Movie-shot-in-Country and Award-awarded-in-Country. As mentioned, these latter links may be least useful for automatic relationship detection.

### Terrorism data

Relational data about world-wide terrorist events is available,<sup>1</sup> as well as ontologies describing the organization of this data (Niles & Pease 2001). From this data we constructed an ontology and semantic graph. The 59 node types are shown in Table 2. The ontology is shown in Figure 5 as an adjacency matrix. The semantic graph contains 2366 nodes.

Figures 6 and 7 plot the average number of neighbors per type and the disparity of connected types, respectively. Er-

<sup>1</sup>Data available at <http://ontology.teknnowledge.com>.

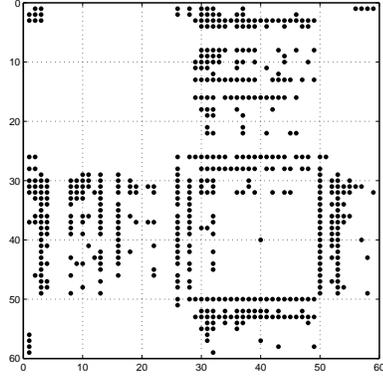


Figure 5: Adjacency matrix for the terrorism ontology. The matrix is used to determine which node types are allowed to link to a given type.

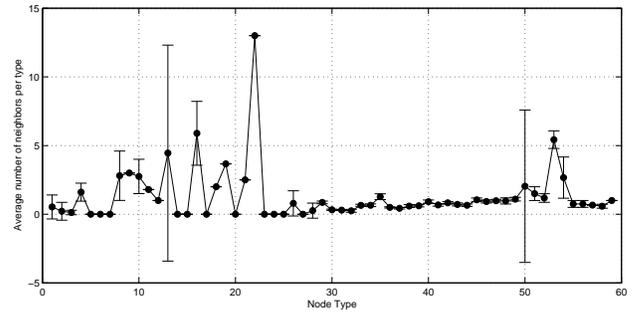


Figure 6: Terrorism data: average number of neighbors per type,  $m_\alpha$ . Each error bar is of length  $\sigma_\alpha^k$  on each side of the average.

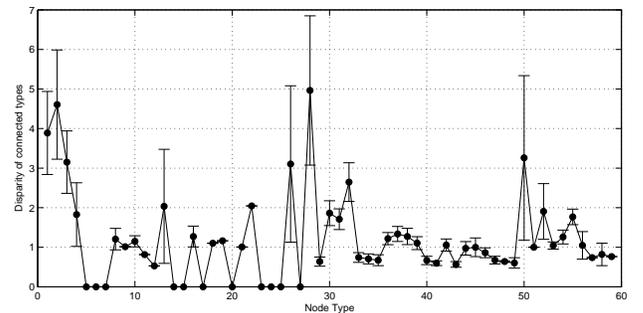


Figure 7: Terrorism data: disparity of connected types,  $R(\alpha)$ . Each error bar is of length  $\sigma_\alpha^R$  on each side.

ror bars are used to show the dispersion of the quantities. We consider that frequencies of 50 or more in this data set are statistically significant. Thus, we consider types 1, 2, 3, 28, 30, 31, 32, 36, 37, 42, and 50. For all these types, the average number of neighbors per type is small. The types, however, can be separated by their disparity. Types 1, 2, 3, 28, and 50 have high disparity, i.e., they are connected to many different types. This is consistent with nodes of types 1, 2, and 3 being of type “location,” nodes of type 28 being of type “terrorist organization,” and nodes of type 50 being of type “humber.” The remaining types are types of attacks and are not particularly correlated with any other node types (given the numbers of each node type). We note in this case that semantically similar node types have similar values of  $m_\alpha$  and  $R(\alpha)$ .

## Conclusion

This paper reveals some of the knowledge representation issues associated with semantic graphs. Ideas from the field of complex networks have been applied and generalized to semantic graphs. For example, transitivity may be used to determine the relevance of edge types for relationship detection.

We have defined several measures for statistically characterizing node types. These quantities take into account the ontology which specifies the permitted connections in the semantic graph. Many other important measures can be defined, such as correlations with attribute values (Jensen & Neville 2002), which was not covered in this paper. These and other tools can be useful to help design ontologies and semantic graphs for knowledge representation.

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